3/12/18

"Experience enables you to recognize a mistake when you make it again."-Franklin P. Jones

HW:

AIM: What is an Anti-Derivative?

Warm Up:

Find $\frac{dy}{dx}$ for each of the following:

1) $y = x^2$

$\frac{dy}{dx} = 2x$

2) $y = 2x^3$

$\frac{dy}{dx} = 6x^2$

3) $y = 3x^6$

$\frac{dy}{dx} = 18x^5$

4) $y = ax^n$

"a" is a constant

$\frac{dy}{dx} = anx^{n-1}$

Find $f(x)$ for each of the following:

5) $f'(x) = x^3 \longrightarrow f(x) = \frac{1}{4}x^4$

6) $f'(x) = x^7 \longrightarrow f(x) = \frac{1}{8}x^8$

7) $f'(x) = 2x^8 \longrightarrow f(x) = \frac{2}{9}x^9$

8) $f'(x) = ax^n \longrightarrow f(x) = \frac{a}{n+1}x^{n+1}$

"a" is a constant

$f(x)$ is called the anti-derivative of $f'(x)$

$F(x) \longrightarrow$ Antiderivative of $f(x)$

$f(x) \longrightarrow$ derivative of $F(x)$
also antiderivative of $f'(x)$

$f'(x) \longrightarrow$ derivative of $f(x)$
also antiderivative of $f''(x)$

Find $f'(x)$ for each of the following:

9) $f(x) = 3x^2 + 2 \longrightarrow f'(x) = 6x$

10) $f(x) = 3x^2 + 9 \longrightarrow f'(x) = 6x$

Each function has a family of functions that represent the Anti-derivative.

$3x^2+2$ and $3x^2+9$ are members of the family of functions whose derivative is $6x$.

$$f(x) = ax^n \longrightarrow F(x) = \frac{a}{n+1}x^{n+1} + C$$

where "c" is a constant
"constant of integration"

11) If $f(x) = 4x^5 - 6x^1 + 4x^0$ and $\underbrace{F(0) = -1}_{\text{initial condition}}$

find $F(x)$.

(Anti-derivative)

$$F(x) = \frac{4}{5+1}x^{5+1} - \frac{6}{1+1}x^{1+1} + \frac{4}{0+1}x^{0+1} + c$$

$$F(x) = \frac{4}{6}x^6 - \frac{6}{2}x^2 + \frac{4}{1}x^1 + c$$

$$F(x) = \frac{2}{3}x^6 - 3x^2 + 4x + c$$ ← General Solution

Plug in 0 for x
and -1 for F(x)

$$-1 = \frac{2}{3}(0)^6 - 3(0)^2 + 4(0) + c$$

$$-1 = c$$

$$F(x) = \frac{2}{3}x^6 - 3x^2 + 4x - 1$$ ← Specific solution that satisfies the initial condition

12) <u>Given:</u> $f'(x) = 8x^3 + 12x + 3$, $\underbrace{f(1) = 6}_{\text{initial condition}}$

<u>Find:</u> $f(x)$

$f(x) = \frac{8}{4}x^4 + \frac{12}{2}x^2 + \frac{3}{1}x^1 + c$ ← General Solutions

$f(x) = 2x^4 + 6x^2 + 3x + c$

$6 = 2(1)^4 + 6(1)^2 + 3(1) + c$

$6 = 2 + 6 + 3 + c$

$6 = 11 + c$

$-5 = c$

$\boxed{f(x) = 2x^4 + 6x^2 + 3x - 5}$ ← Specific solution.

13) Given: $f''(x) = \sqrt{x} + 3$, $\underbrace{f'(0) = -3}_{\text{initial condition}}$

Find: $f'(x)$

$$f''(x) = x^{1/2} + 3$$

$$f'(x) = \frac{1}{3/2} x^{3/2} + 3x + c$$

$$f'(x) = \frac{2}{3} x^{3/2} + 3x + c$$

$$-3 = \frac{2}{3}(0)^{3/2} + 3(0) + c$$

$$-3 = 0 + c$$

$$-3 = c$$

$$\boxed{f'(x) = \frac{2}{3} x^{3/2} + 3x - 3}$$

14) Given: $f'(x) = 24x^2 + 2x + 10$, $f(1) = -3$

$F(1) = 5$

Find: $F(x)$

$$f(x) = \frac{24}{3}x^3 + \frac{2}{2}x^2 + 10x + c$$

$$f(x) = 8x^3 + x^2 + 10x + c$$

$$-3 = 8(1)^3 + 1^2 + 10(1) + c$$

$$-3 = 19 + c$$

$$-22 = c$$

$$f(x) = 8x^3 + x^2 + 10x - 22$$

$$F(x) = \frac{8}{4}x^4 + \frac{1}{3}x^3 + \frac{10}{2}x^2 - 22x + d$$

$$F(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + d$$

$$5 = 2(1)^4 + \frac{1}{3}(1)^3 + 5(1)^2 - 22(1) + d$$

$$5 = 2 + \frac{1}{3} + 5 - 22 + d$$

$$5 = -\frac{44}{3} + d$$

$$+\frac{44}{3} \qquad +\frac{44}{3}$$

$$\frac{59}{3} = d$$

$$\boxed{F(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}}$$